ENG 301
Engineering Mathematics III

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LINEAR ALGEBRA

SYSTEMS OF LINEAR EQUATIONS

WRITING SYSTEMS

20X + 35Y =70

$$1x_1 + x_2 = 5$$
$$3x_1 - 2x_2 = 10$$

$$\begin{bmatrix} 1 & 1 \\ 3 & -2 \end{bmatrix}$$

Coefficient matrix

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 3 & -2 & 10 \end{array}\right]$$

Augmented Matrix

WRITING SYSTEMS

$$2x_1 + 2x_2 = \cancel{5}\ 10$$
$$+\ 3x_1 - 2x_2 = 10$$
$$5x_1 = 20$$
$$\frac{5x_1}{5} = \frac{20}{5} \qquad x_1 = 4$$
$$x_1 + x_2$$

$$\begin{bmatrix} 1 & 1 \\ 3 & -2 \end{bmatrix}$$

Coefficient matrix

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 3 & -2 & 10 \end{array}\right]$$

Augmented Matrix

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 3 & -2 & 10 \end{array}\right] \xrightarrow{2R_1 + R_2 \to R_1} \left[\begin{array}{cc|c} 5 & 0 & 20 \\ 3 & -2 & 10 \end{array}\right]$$

$$\xrightarrow{\frac{1}{5}R_1 \to R_1} \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 3 & -2 & 10 \end{array}\right]$$

WRITING SYSTEMS

$$2x_1 + 2x_2 = \cancel{5}\ 10$$
$$+\ 3x_1 - 2x_2 = 10$$
$$5x_1 = 20$$
$$\frac{5x_1}{5} = \frac{20}{5} \qquad x_1 = 4$$
$$x_1 + x_2 = 5$$
$$4 + x_2 = 5$$
$$x_2 = 1$$

$$\begin{bmatrix} 1 & 1 \\ 3 & -2 \end{bmatrix}$$

Coefficient matrix

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 3 & -2 & 10 \end{array}\right]$$

Augmented Matrix

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 3 & -2 & 10 \end{array}\right] \xrightarrow{2R_1 + R_2 \to R_1} \left[\begin{array}{cc|c} 5 & 0 & 20 \\ 3 & -2 & 10 \end{array}\right]$$

$$\xrightarrow{\frac{1}{5}R_1 \to R_1} \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 3 & -2 & 10 \end{array}\right] \xrightarrow{-3R_1 + R_2 \to R_2} \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & -2 & -2 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{2}R_2 \to R_2} \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 1 \end{array}\right] \to \begin{array}{l} 1x_1 + \cancel{0x_2} = 4 \\ \cancel{0x_1 +} 1x_2 = 1 \end{array} \qquad \begin{array}{l} x_1 = 4 \\ x_2 = 1 \end{array}$$

TERMINOLOGY

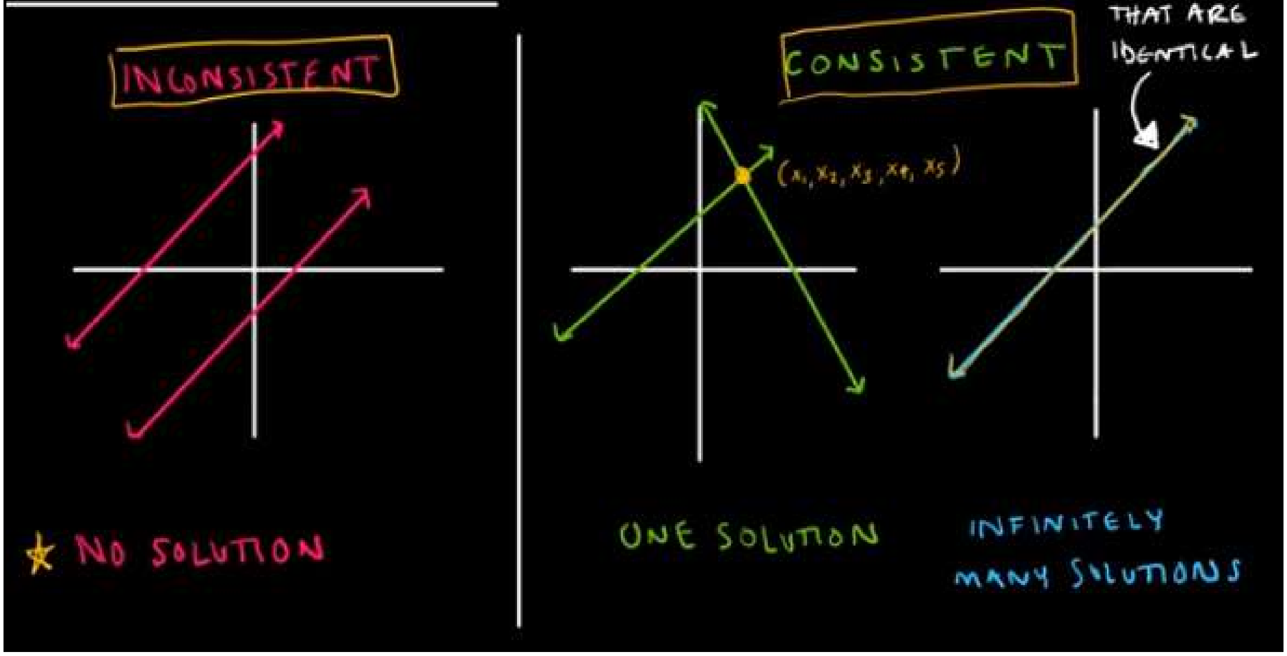
LINEAR EQUATION - An equation that can be written as $a_1x_1 + a_2x_2 + a_3x_3 + \dots + a_nx_n = b$ where $a_1, a_2, \dots a_n, b$ are real or complex numbers.

SYSTEM OF LINEAR EQUATIONS - A collection of two or more linear equations using the same variables.

SOLUTION - A list of numbers $(s_1, s_2, s_3 \dots)$ that makes each equation in the system true when substituted for $x_1, x_2, x_3 \dots$ respectively.

SOLUTION SET - The set of all possible solutions to a system.

TYPES OF SYSTEMS



PRACTICE

SOLVE THE GIVEN SYSTEM OF EQUATIONS USING ELIMINATION.

$$x_1 - 2x_2 + x_3 = 0$$
$$2x_2 - 8x_3 = 8$$
$$-4x_1 + 5x_2 + 9x_3 = -9$$

$(29, 16, 3)$

PRACTICE

SOLVE THE GIVEN SYSTEM OF EQUATIONS USING ELIMINATION.

$$4(x_1 - 2x_2 + x_3 = 0)$$
$$2x_2 - 8x_3 = 8$$
$$-4x_1 + 5x_2 + 9x_3 = -9$$

$2x_2 - 8x_3 = 8$

$$4x_1 - 8x_2 + 4_3 = 0$$
$$-4x_1 + 5x_2 + 9x_3 = -9$$

PRACTICE

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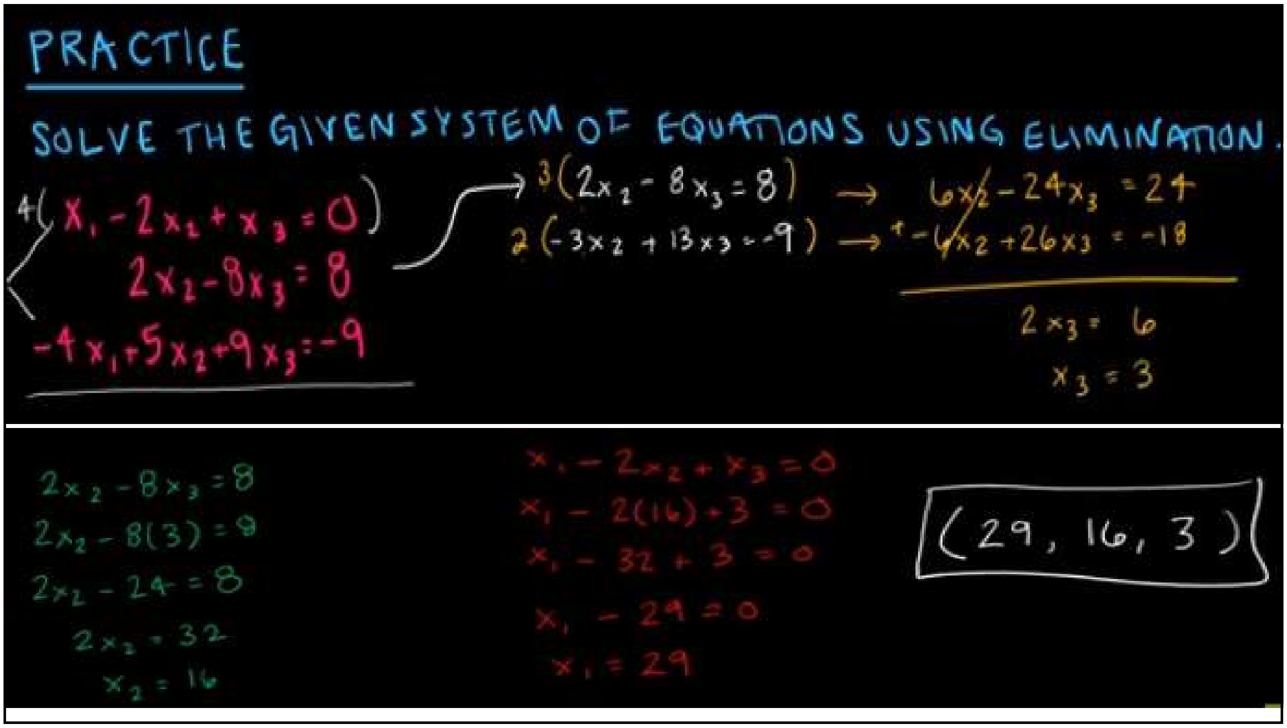
$2x_2 - 8x_3 = 8$

$-3x_2 + 13x_3 = -9$

$$4x_1 - 8x_2 + 4_3 = 0$$
$$+ \; -x_1 + 5x_2 + 9x_3 = -9$$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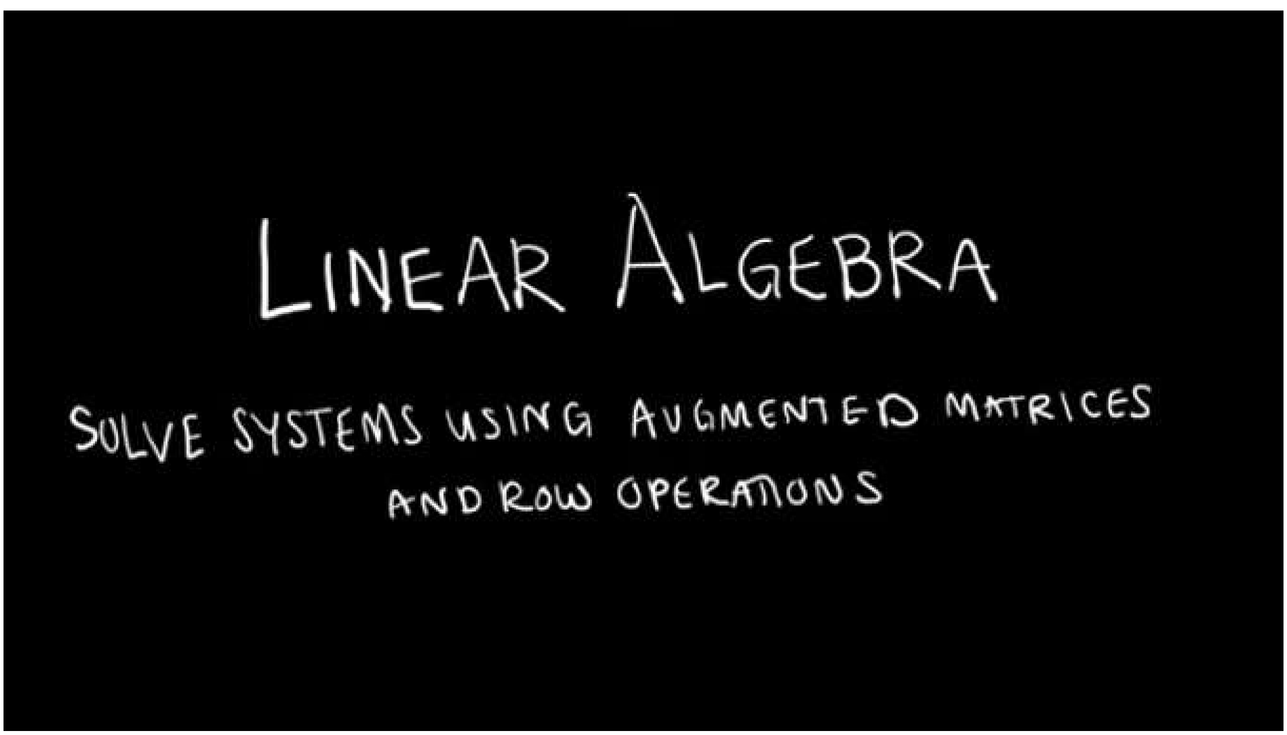
$3(2x_2 - 8x_3 = 8) \rightarrow 6x_2 - 24x_3 = 24$
$2(-3x_2 + 13x_3 = -9) \rightarrow + -6x_2 + 26x_3 = -18$
$2x_3 = 6$
$x_3 = 3$
$2x_2 - 8x_3 = 8$
$2x_2 - 8(3) = 8$
$2x_2 - 24 = 8$
$2x_2 = 32$
$x_2 = 16$
$x_1 - 2x_2 + x_3 = 0$
$x_1 - 2(16) + 3 = 0$
$x_1 - 32 + 3 = 0$
$x_1 - 29 = 0$
$x_1 = 29$
$(29, 16, 3)$


LINEAR ALGEBRA
SOLVE SYSTEMS USING AUGMENTED MATRICES
AND ROW OPERATIONS

ROW OPERATIONS

REPLACEMENT - REPLACE ONE ROW BY THE SUM OF ITSELF AND A MULTIPLE OF ANOTHER ROW

$$\left[\begin{array}{cc|c} 2 & 4 & 8 \\ 1 & 0 & 9 \end{array}\right]$$

INTERCHANGE - INTERCHANGE (SWAP) TWO ROWS

$$\left[\begin{array}{cc|c} 2 & 4 & 8 \\ 1 & 0 & 9 \end{array}\right]$$

SCALING - MULTIPLY A ROW BY A NON-ZERO CONSTANT

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ROW OPERATIONS

REPLACEMENT - REPLACE ONE ROW BY THE SUM OF ITSELF AND A MULTIPLE OF ANOTHER ROW

$$\left[\begin{array}{cc|c} 2 & 4 & 8 \\ 1 & 0 & 9 \end{array}\right] \xrightarrow{-2R_2 + R_1 \to R_2} \left[\begin{array}{cc|c} 2 & 4 & 8 \\ 0 & 4 & -10 \end{array}\right]$$

$-2 \quad 0 \quad -18$

INTERCHANGE - INTERCHANGE (SWAP) TWO ROWS

$$\left[\begin{array}{cc|c} 2 & 4 & 8 \\ 1 & 0 & 9 \end{array}\right] \xrightarrow{R_1 \Leftrightarrow R_2} \left[\begin{array}{cc|c} 1 & 0 & 9 \\ 2 & 4 & 8 \end{array}\right]$$

SCALING - MULTIPLY A ROW BY A NON-ZERO CONSTANT

$$\left[\begin{array}{cc|c} 2 & 4 & 8 \\ 1 & 0 & 9 \end{array}\right] \xrightarrow{\frac{1}{2}R_1 \to R_1} \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 1 & 0 & 9 \end{array}\right]$$

SOLVE THE SYSTEM (AGAIN)

THIS TIME USE AN AUGMENTED MATRIX AND ROW OPERATIONS

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ -4x_1 \quad x_2 + \quad x_3 &= -9 \end{aligned} \qquad \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & -9 \end{array}\right]$$

(29, 16, 3)

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THIS TIME USE AN AUGMENTED MATRIX AND ROW OPERATIONS

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ -4x_1 \quad x_2 + \quad x_3 &= -9 \end{aligned} \qquad \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & -9 \end{array}\right]$$

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$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ -4x_1 \quad x_2 + \quad x_3 &= -9 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & \boxed{2} & -8 & 8 \\ \boxed{-4} & \boxed{5} & 9 & -9 \end{array}\right] \xrightarrow{4R_1 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & -3 & 13 & -9 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & -3 & 13 & -9 \end{array}\right] \xrightarrow{3R_2 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Back Subst.

$$\begin{aligned} x_1 - 2x_2 + 1x_3 &= 0 \\ 1x_2 - 4x_3 &= 4 \\ 1x_3 &= 3 \end{aligned} \quad \rightarrow \quad \begin{aligned} x_2 - 4(3) &= 4 \\ x_2 - 12 &= 4 \\ x_2 &= 16 \end{aligned} \qquad \begin{aligned} x_1 - 2(16) + 3 &= 0 \\ x_1 - 32 + 3 &= 0 \\ x_1 - 29 &= 0 \\ x_1 &= 29 \end{aligned}$$

$(29, 16, 3)$

Try to complete these solutions using the row operations from here!

Hint? ...

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

EXISTENCE AND UNIQUENESS

DETERMINE IF THE SYSTEM IS CONSISTENT (DOES A SOLUTION EXIST?)

IF SO, DETERMINE IF THE SOLUTION IS UNIQUE (JUST ONE SOLUTION?)

$$\begin{aligned} x_2 - 4x_3 &= 8 \\ 2x_1 - 3x_2 + 2x_3 &= 1 \\ 4x_1 - 8x_2 + 12x_3 &= 1 \end{aligned}$$

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$$\left[\begin{array}{ccc|c} 0 & 1 & -4 & 8 \\ 2 & -3 & 2 & 1 \\ 4 & -8 & 12 & 1 \end{array}\right]$$

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$$x_2 - 4x_3 = 8$$
$$2x_1 - 3x_2 + 2x_3 = 1$$
$$4x_1 - 8x_2 + 12x_3 = 1$$

$$\left[\begin{array}{ccc|c} 0 & 1 & -4 & 8 \\ 2 & -3 & 2 & 1 \\ 4 & -8 & 12 & 1 \end{array}\right] \xrightarrow{R_1 \Leftrightarrow R_2} \left[\begin{array}{ccc|c} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 4 & -8 & 12 & 1 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_1 \to R_2} \left[\begin{array}{ccc|c} 1 & -3/2 & 1 & 1/2 \\ 0 & 1 & -4 & 8 \\ 4 & -8 & 12 & 1 \end{array}\right] \xrightarrow{-4R_1 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & -3/2 & 1 & 1/2 \\ 0 & 1 & -4 & 8 \\ 0 & -2 & 8 & -1 \end{array}\right] \xrightarrow{2R_2 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & -3/2 & 1 & 1/2 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 15 \end{array}\right]$$

$$0x_1 + 0x_2 + 0x_3 = 15$$

INCONSISTENT